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ABSTRACT

To ascertain if children's use of manipulatives and/or the teacher's presentation of the problem in parts (chunking) affect children's ability to model and/or solve addition and/or subtraction problems, 146 first-grade subjects were administered the Houston Addition and Subtraction Problem-Solving Test which provided accuracy and modeling scores for two addition and three subtraction problem types. The test was presented in holistic (not chunked) and parsimonious (chunked) forms. Results indicated that chunking significantly affects children's ability to solve nontransformational addition problems. The interaction of chunking and modeling significantly improved the subjects' ability to solve take-away subtraction problems. (Author)

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An Analysis of the Effects of the Use of Manipulatives and Problem "Chunking" on First Grade Children's Addition and Subtraction Problem Solving Modeling and Accuracy

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To ascertain if the children's use of manipulatives and/or the teacher's presentation of the problem in parts (chunking) effect children's ability to model and/or solve addition and/or subtraction problems, 146 first grade subjects were administered the Houston Addition and Subtraction Problem Solving Test which provided accuracy and modeling scores for two addition and three subtraction problem types. The test was presented in wholistic (not chunked) and parsimonious (chunked) forms. Results indicated that chunking significantly effects the children's ability to solve non-transformational addition problems. The interaction of chunking and modeling significantly improved the subjects' ability to solve take-away subtraction problems.

An Analysis of the Effects of the Use of Manipulatives and Problem "Chunking" on First Grade Children's Addition and Subtraction Problem Solving Modeling and Accuracy.

Objective

This study was conducted to ascertain if the children's use of manipulatives and/or the teacher's presentation of the problem in parts (chunking) effect children's ability to model and/or solve addition and/or subtraction problems.

Theoretical Background

Children's mathematics problem solving abilities and strategies have been critically examined by many scholors. They have documented the effects of reading skills (Vanderlinde, 1964), problem structure (Burns and Yonally, 1964), familiarity with problem operants (Steffe, 1967), and degree of involvement of the student (Chase, 1960) on the problem solving abilities of students. Other researchers have established that students' problem solving abilities may be enhanced when models, correction techniques, intermittent praise, graduated difficulty problems, drawings, and peer problems are employed (Riedesel and Burns, 1973). Few, however, have delt with the effects of the developmental construct of conservation upon mathematical reasoning ability.

The variable of conservation of numerousness was introduced to the mathematics education community from the translated writings of Jean Piaget (1965), the text by Flavell (1963), and the research of Elkind (1961), Dodwell (1960), and Wohlwill (1962). Research studies related to this variable began with those by Van Engen and Steffe (1966) on first grade addition problem solving skills, LeBlanc (1968) on subtraction problem solving skills, Steffe (1970) on addition, and Steffe and Johnson (1971) on addition and subtraction. In these studies, the childrens' abilities to make quantitative comparisons (either gross, intensive, or extensive) was treated as an independent variable in studying problem solving abilities.

Previous researchers have reported significant differences between the problem solving achievement of first graders who conserve and those who do not conserve. These researchers also found significant differences in problem sovling achievement related to problem characteristics. In addition, it was found that problems which involved a transformation* were significantly more difficult than those which did not involve a transformation. In subtraction, the "take-away" subtraction problems without transformations were found to be significantly more difficult that "take-away" subtraction problems with transformations.

While previous studies were carefully conducted and subjected to rigorous statistical analysis, a major question remains unanswered. Since these addition and subtraction

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*A <u>transformation</u> is defined as an implied physical movement of objects associated with an addition problem describing joining of sets, e.g., "Mary has three dolls and her mother gave her two more." <u>No transformation</u> is a static condition, e.g., "John has two frogs and David has three frogs."

studies used models in several distinctive forms and presented the problems in either a whole problem or "part-at-a-time" form, to what extent were the findings attributable to the form of model used and the type of presentation employed? This is a particularly cogent question when one analyzes instruction in the first grade classroom where problems are frequently verbally modeled and presented in a wholistic fashion.

The use of manipulatives has been the subject of several studies. Steffe (1968, 1970), Le Blanc (1968), and Steffe and Johnson (1970, 1971) found a significant main effect in each study in favor of subjects who had manipulative treatments. The subject's success has been attributed in part to the fact that first grade children, aged 5 to 7 years, who are beginning to conserve numerousness are given perceptual cues by manipulative operations which assist tham in maintaining the integrity of the initial set(s).

In many skill areas the use of whole as opposed to part learning has been shown to be associated with skills acquisition (Knapp and Dixon, 1952; Lanellen, 1951; Purdy and Stallard, 1967; Shay, 1934; E. Young, 1965). In the instance of learning addition and subtraction the case for whole (not chunked) learning is less clear, however, than in the reading and physical education instances reviewed. For, the subject's ability to accurately perceive the sets involved in addition and subtraction may be facilitated by their singular presentation (chunking).

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Sampling

One hundred forty six (146) kindergarten children were randomly selected from one Area of the Houston Independent School District of Houston, Texas. Becuase of its geographic distribution, cutting across the urban and suburban sections of the city, it was felt that the Area contained a representative cross-section of ethnic and socio-economic groups. There were 3,061 first grade children in the Area.

Instrumentation and Procedures

All subjects were administered the Houston Addition and Subtraction Problem Solving Test (HASPST). The HASPST consisted of 11 items: 6 subtraction (2 each of take-away, comparison and additive), 4 addition (2 each of transformation and non-transformation), and one transition item. Before each individual administration, the order, addition then subtraction or vice versa, was determined by the flip of a coin. The order of the six subtraction questions and four addition questions was also determined randomly for each administration. The transition item was included because a pilot study revealed that responses to the first item in a sequence at the point of operation change was often characterized by a continuation of the first operation; this item always agreed in problem type with the last randomly ordered problem in the ten evaluated items; performance on the transition item was not used in the analysis.

The test was presented in two forms (chunked, not chunked) with three types of manipulative usage (required, optional, none). In each administration the subject and researcher were seated across from one another at a table. In all but the no manipulative treatment, in front of the subject was a small basket of chips and two eight inch, cardboard dolls. To provide for maximal success of the subjects, the subjects named the dolls, e.g., Michelle and Ramos, and the chips, e.g., hamburgers. All eleven questions then used the names and label provided by each subject. For the "chunked" treatment, each question was presented one-sentence-at-a-time with a pause for the subject to "act out" the sentence with the manipulatives provided: subjects' use of manipulatives was required for the required treatment and probing questions were used to obtain responses not spontaneously provided or unclear to the researcher. Each subject received 0 or 1 point for the accuracy of each response (the criterion was a correct response ±1). Each subject received 0, 1, or 2 points for the modeling behaviors exhibited relative to each problem according to criteria specified for each of the five problem types (see Underhill and Shores, 1975). It was necessary to account for the nesting of variables within each state (accuracy and modeling) of the dependent variable of problem solving, for problem solving occured in addition and subtraction cases with two and three types of problems in each case, respectively. The data, thus, took the form presented in figure 1. With the nested design, Insert Figure 1 About Here

it was possible to trace significant main effect findings to the specific type of problem which contributed to the main effect finding. Modeling scores were, or course, unavailable for the "no modeling" treatment and the n was reduced for the "optional modeling" treatment.

Analysis

Prior to testing the independent variables, it was necessary to ascertain the interpretability ot the measures. A principle component factor analysis of the Addition-Subtraction Problem Solving Test (Underhill and Shores, 1975) revealed that subjects responded consistently to combined addition problems and the three distinct subtraction types when accuracy scores were used. However, when modeling scores were analyzed, the additive subtraction items combined with one type of addition.

Initial multiple linear regression analysis of the dependent variables using sex and age (in months) as criterion variables confirmed that variance due to age must be statistically removed from subsequent analyses due to its relationship to both chunking and modeling.

The nested analysis of covariance of the modeling scores was conducted. No significant effects were noted. It must be mentioned that limited data was available on the "optional modeling" treatment.

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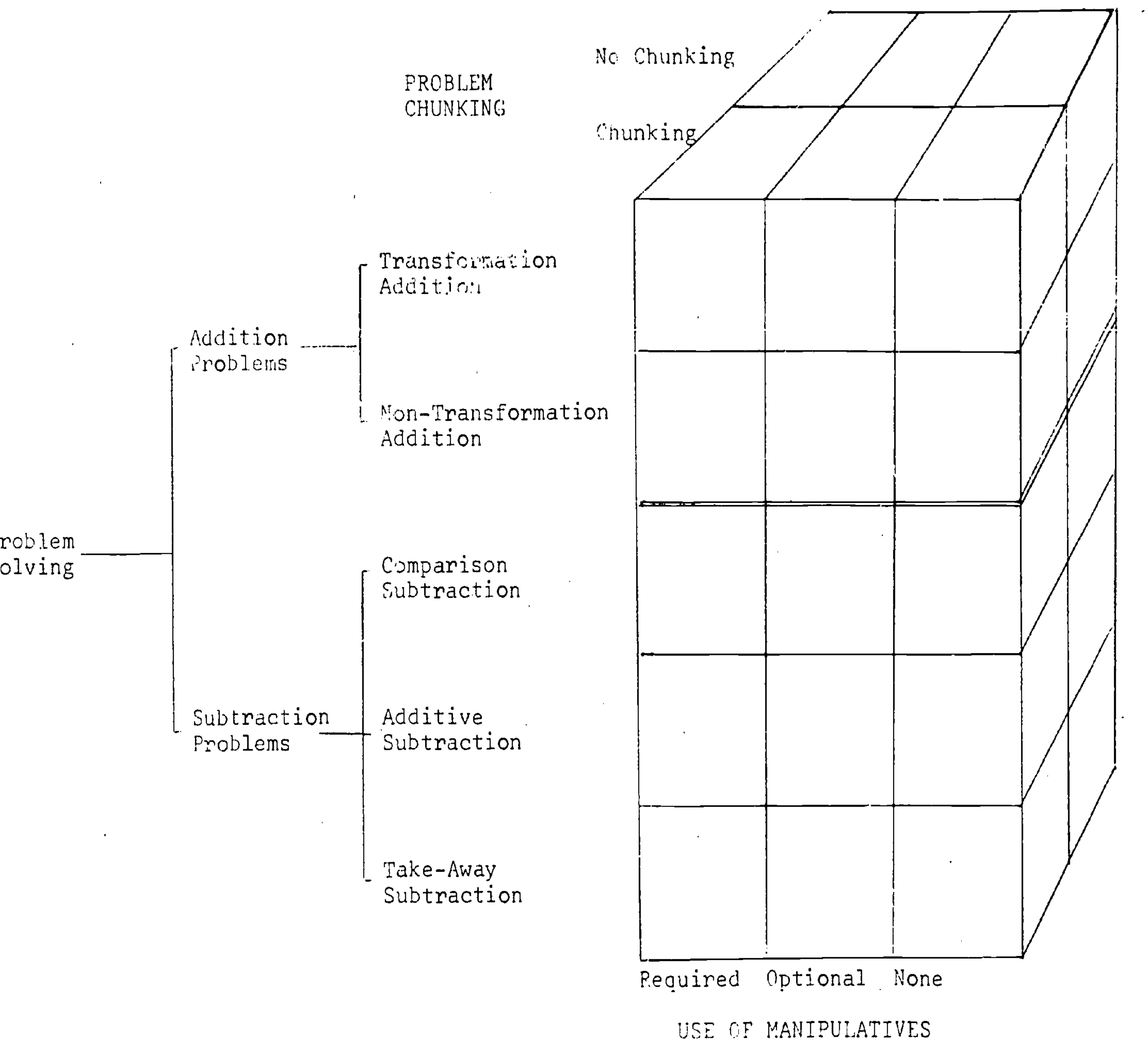

FIG. 1. Data Utalized in the Study

The nested analysis of covariance of the accuracy scores is summarized in figure 2 and table 1. As figure two indicates, total accuracy scores approached significance on the chunking variable ($p \leq .10$). This finding was traced to the addition accuracy scores ($p \leq .05$) and to the non-transformational scores ($p \leq .05$). The transformational scores approached significance ($p \leq .10$). -Insert FIG-2 & Table 1 About Here-

In a subsequent analysis, the subtraction accuracy scores were found to be significant on the take-away problems on the basis of the interaction of the dependent variables ($p \leq .05$). This significant finding was attributed to the superiority of the required and chunked combination. A significant interaction was approached by the comparison subtraction scores ($p \leq .10$) with highest scores obtained on the required and chunked combination.

Significance of the Findings

The results of this study support the assumption that subjects are aided by being given an opportunity to mentally rehearse the sets presented to them as they solve addition problems. They are not aided by the use of manipulatives contrary to the predominant findings of prior studies of the subject.

That addition is effected rather than subtraction is of great import for the addition problems require the formation of two sets, their union, and a subsequent counting of the joined sets. Subtration, on the other hand, requires that,

Table 1

Summary of Analysis of Covariance of

Accuracy Scores for Use of Manipulatives

and Problem Chunking

Total Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	9.76	2	4.88	.59
Chunking	27.40	1	27.40	3.32*
Interaction	8.38	2	4.19	.51
Error	1130.04	137	8.25	

Total Subtraction Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	3.40	2	1.70	.48
Chunking	3.94	1	3.94	1.10
Interaction	7.61	2	3.80	1.07
Error	488.92	137	3.57	

Total Addition Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	2.21	2	1.11	.52
Chunking	10.56	1	10.56	4.97**
Interaction	.69	2	.35	.16
Error	290.99	137	2.12	

Additive Subtraction Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	1.44	2	.72	.97
Chunking	.35	1	.35	.46
Interaction	3.19	2	1.56	.12
Error	101.72	137	.74	

Comparison Subtraction Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	2.24	2	1.12	1.64
Chunking	2.06	1	2.06	3.03*
Interaction	3.88	2	1.94	2.84*
Error	93.49	137	.68	

Take-Away Subtraction Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	3.56	2	1.68	2.92*
Chunking	.00	1	.00	.00
Interaction	3.38	2	1.68	2.94**
Error	78.71	137	.58	

Table 1 (continued)

Transformation Addition Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	.72	2	.36	.68
Chunking	1.82	1	1.82	3.44*
Interaction	.72	2	.36	.68
Error	72.53	137	.52	

Non-Transformation Addition Score Used as Basis

Source	ss	df	MS	F
Use of Manipulatives	.45	2	.22	.35
Chunking	3.60	1	3.60	5.64**
Interaction	.08	2	.04	.06
Error	87.60	137	.64	

* $p \leq .10$
** $p \leq .05$

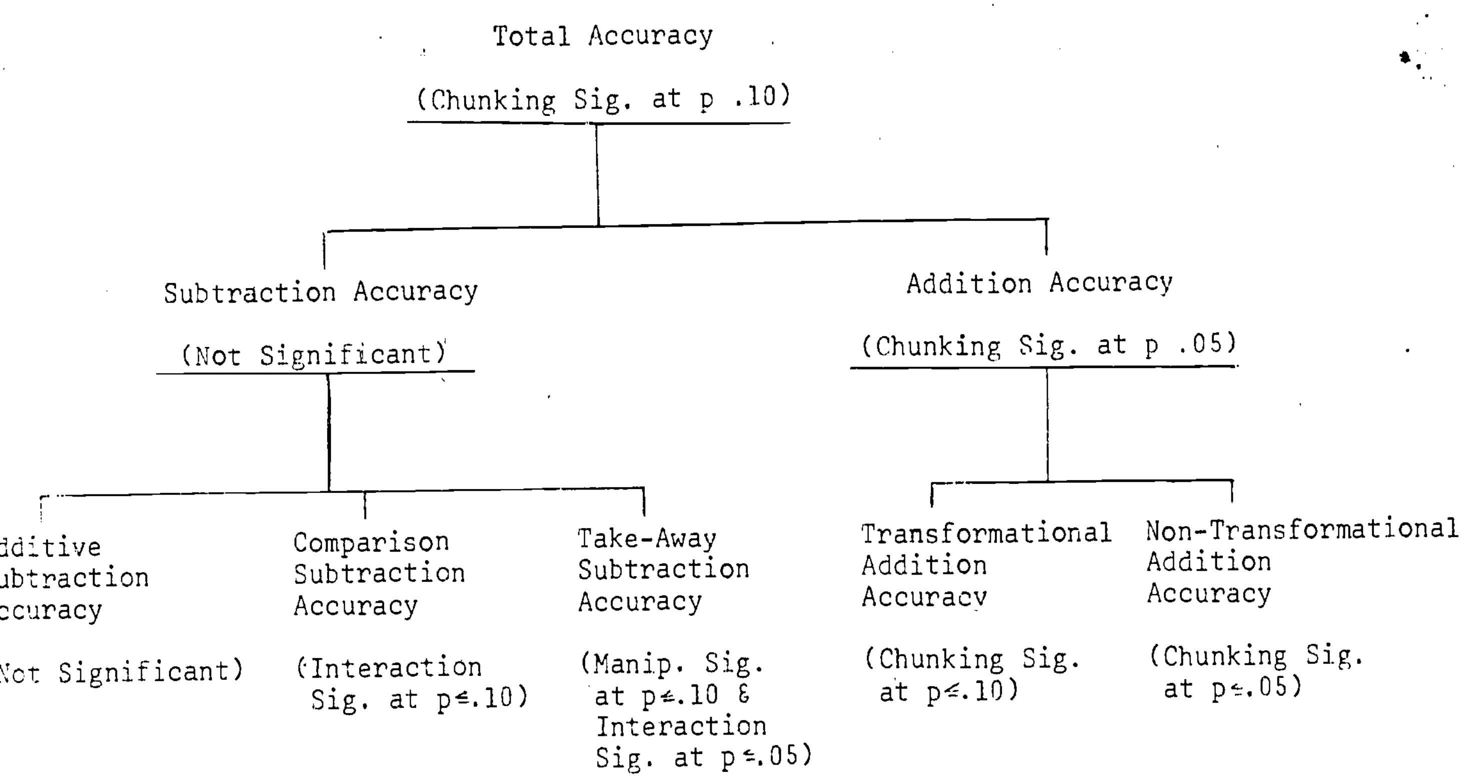

FIG. 2. Summary of Analysis of Covariance of the Accuracy Scores on Chunking and Manipulative Variables

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following the formation of the initial sets, a state of equivalence be achieved (through 1:1 correspondence or some other means) before the resultant set is formed and counted. That chunking assists the former and not the latter may be attributable to the fact that the equivalizing operation is not explicit in the problem statement and, as such, was not rehearsed during the chunking treatment.

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